Chapter 6. Squares and Square Roots

Question 1

Which of the following numbers are perfect squares? 11, 12, 16, 32, 36

Solution:

11 is not a perfect square because it is a prime number.
12 is not a perfect square because its units digit is 2.
16 is a perfect square because 16 = 4 x 4.
32 is not a perfect square because its units digit is 2.
36 is a perfect square because 36 = 6 x 6.

Question 2

Which of the following perfect squares are squares of even numbers? 121, 225, 256, 1296, 6561

Solution:

256 and 1296 are squares of even numbers because they are even numbers.
121, 225 and 6561 are not squares of even numbers because they are odd numbers.

Question 3

Which of the following numbers are perfect squares? 100, 1000, 330550, 12345600000

Solution:

100 is a perfect square because the number of zeros in the end is even.
1000, 330550 and 12345600000 are not perfect squares because the number of zeros in the end is odd.

Question 4

Write down the correct number in the box: [For every natural number n, we have $(n + 1)^2 - n^2 = (n + 1 - n)(n + 1 + n) = (n + 1) + n$.]

(a) $23^2 - 22^2$ = _____
(b) $101^2 - 100^2$ = _____
(c) $551^2 - 550^2$ = _____

Solution:

(a) $23^2 - 22^2$ =23 + 22 = 45
(b) $101^2 - 100^2$ =101 + 100 = 201
(c) $551^2 - 550^2$ =551 + 550 = 1101

Question 5

Write down the correct number in each box:

(a) 65^2 = 6 x 7(100) + 5^2 = 4200 + 25 =_____

(b) 75^2 = 7 x 8(100) + 5^2 = 5600 + 25 =_____

Solution:

(a) 65^2 = 6x 7(100) + 5^2 = 4200 + 25 = 4225

(b) 75^2 = 7 x 8(100) + 5^2 = 5600 + 25 = 5625

Question 6

Which of the following triplets are Pythagorean?

(i) (1, 2, 3) (ii) (3, 4, 5) (iii) (6, 8, 10)

[We know that the three natural numbers m, n, p are said to be Pythagorean triplets if $m^2 + n^2 = p^2$.]

Solution:

(i) $1^2 + 2^2 = 1 + 4 = 5 \neq 3^2$

(ii) $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

(iii) $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

Therefore, (3, 4, 5) and (6, 8, 10) are Pythagorean triplets.

Question 7

Which of the following triplets are Pythagorean?

(i) (1, 1, 1) (ii) (2, 2, 3)

[We know that the three natural numbers m, n, p are said to be Pythagorean triplets if $m^2 + n^2 = p^2$.]

Solution:

(i) $1^2 + 1^2 = 1 + 1 = 2 \neq 1^2$

(ii) $2^2 + 2^2 = 4 + 4 = 8 \neq 3^2$

Therefore there are no Pythagorean triplets.

Question 8

Observe a pattern in the following and find the missing numbers:

$$121 = \frac{22^2}{1+2+1}$$

$$12321 = \frac{(333)^2}{1+2+3+2+1}$$

$$1234321 = \_\_\_\_\_$$

$$123454321 = \_\_\_\_\_$$

$$12345654321 = \_\_\_\_\_$$

$$1234567654321 = \_\_\_\_\_$$

$$123456787654321 = \_\_\_\_\_$$

$$12345678987654321 = \frac{(999999999)^2}{1+2+3+4+5+6+7+8+9+8+7+6+5+4+3+2+1}$$

Solution:

$$121 = \frac{22^2}{1+2+1}$$

$$12321 = \frac{(333)^2}{1+2+3+2+1}$$

$$1234321 = \frac{(4444)^2}{1+2+3+4+3+2+1}$$

$$123454321 = \frac{(55555)^2}{1+2+3+4+5+4+3+2+1}$$

$$12345654321 = \frac{(666666)^2}{1+2+3+4+5+6+5+4+3+2+1}$$

$$1234567654321 = \frac{(7777777)^2}{1+2+3+4+5+6+7+6+5+4+3+2+1}$$

$$123456787654321 = \frac{(88888888)^2}{1+2+3+4+5+6+7+8+7+6+5+4+3+2+1}$$

Question 9

Use calculator to find the squares of all natural numbers between 80 and 90.

Solution:

The squares of all natural between 80 and 90 are as follows:

$81^2 = 6561$

$82^2 = 6724$

$83^2 = 6889$

$84^2 = 7056$

$85^2 = 7225$

$86^2 = 7396$

$87^2 = 7569$

$88^2 = 7744$

$89^2 = 7921$

Question 10

Guess and verify the square root of:

a) 25 b) 64 c) 81 d) 100 e) 169

f) 225 g) 400 h) 4900 i). 39 × 39 j) 320 × 320

Solution:

a) $25 = 5^2$

∴ The square root of 25 is 5.

b) $64 = 8^2$

∴ The square root of 64 is 8.

c) $81 = 9^2$

∴ The square root of 81 is 9.

d)$100 = 10^2$

∴ The square root of 100 is 10.

e)$169 = 13^2$

∴ The square root of 169 is 13.

f) $225 = 15^2$

∴ The square root of 225 is 15.

g)$400 = 4 \times 100 = 2^2 \times 10^2 = 20^2$

∴ The square root of 400 is 20.

h) $4900 = 49 \times 100 = 7^2 \times 10^2 = 70^2$

∴ The square root of 4900 is 70.

i) The square root of 39 x 39 is 39.

j) The square root of 320 x 320 is 320.

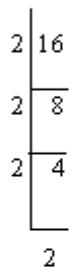
Question 11

Find the square root of the following by means of factors:

(a) 16 (b) 196 (c) 529 (d) 400 (e) 1764 (f) 4096
(g) 7744 (h) 11664 (i) 4900 (j) 47089 (k) 298116

Solution:

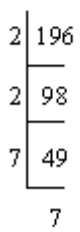
a)



$16 = 2 \times 2 \times 2 \times 2 = (2^2 \times 2^2) = (2\ 2)^2$

∴ The square root of 16 = 2 ´ 2= 4

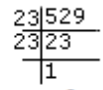
b)



$196 = 2 \times 2 \times 7 \times 7 = (2^2 \times 7^2) = (2 \times 7)^2$

∴ The square root of 196 = 2 × 7= 14

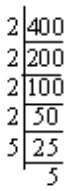
c)



$529 = 2^3 \times 2^3 = 23^2$

∴ The square root of 529 = 23

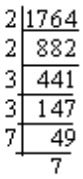
d)



$400 = 2\times2\times2\times2\times5\times5 = (2^2 \times 2^2 \times 5^2) = (2 \times 2 \times 5)^2$

∴ The square root of 400 = 2×2×5= 20

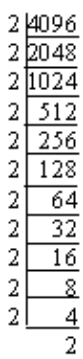
e) 1764 = 2×2×3×3×7×7



$1764 = 2\times2\times3\times3\times7\times7 = (2^2 \times 3^2 \times 7^2) = (2 \times 3 \times 7)^2$

∴ The square root of 1764 = 2×3×7= 42

f)

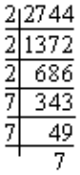


$4096 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2 = (2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2)$

$= (2 \times 2 \times 2 \times 2 \times 2 \times 2)^2$

∴ The square root of 4096=2×2×2×2×2×2= 64

g)



$7744 = 2\times2\times2\times2\times2\times2\times11\times11 = (2^2 \times 2^2 \times 2^2 \times 11^2) = (2 \times 2 \times 2 \times 11)^2$

∴The square root of 7744 = 2× 2×2×11= 88

h)

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
	7

$11664 = 2\times2\times2\times2\times3\times3\times3\times3\times3\times3 = (2^2 \times 2^2 \times 3^2 \times 3^2 \times 3^2)$

$= (2 \times 2 \times 3 \times 3 \times 3)^2$

∴The square root of 11664=2×2×3×3×3= 108

(i)

2	4900
2	2450
5	1225
5	245
7	49
	7

$4900 = 2\times2\times5\times5\times7\times7 = (2^2 \times 5^2 \times 7^2) = (2 \times 5 \times 7)^2$

∴ The square root of 4900 = 2×5×7= 70

j)

7	47089
7	6727
31	961
	31

$47089 = 7\times7\times31\times31 = (7^2 \times 31^2) = (7 \times 31)^2$

∴The square root of 47089 = 7 × 31 = 217.

k)

2	298116
2	149058
3	74529
3	24843
7	8281
7	1183
13	169
	13

$298116 = 2\times2\times3\times3\times7\times7\times13\times13 = (2^2 \times 3^2 \times 7^2 \times 13^2) = (2 \times 3 \times 7 \times 13)^2$

∴The square root of 298116 = 2×3×7×13 = 546

Question 12

Find the smallest number by which 9408 must be divided so that it becomes a perfect square. Also, find the square root of the perfect square so obtained.

Solution:

2	26244
2	13122
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
	3

9408 = 2×2×2×2×2×2×3×7×7.

The prime factors 2 and 7 occur in pairs.
But prime factor 3 doesn't have a pair.
Therefore, 3 is the smallest number by which 9408
must be divided so that it becomes a perfect square.

∴ Perfect square = 9408,3= 3136
= 2×2×2×2×2×2×7×7

∴ Square root = 2×2×2×7 = 56

Question 13

5929 students are sitting in an auditorium in such a manner that there are as many students in a row as there are rows in the auditorium. How many rows are there in the auditorium?

Solution:

There are as many students in a row as there are rows in the auditorium means

7	5929
7	847
11	121
	11

5929 = 7× 7×11×11

$\sqrt{5929} = \sqrt{7 \times 7 \times 11 \times 11} = 7 \times 11 = 77$rows

∴77 rows are there in the auditorium.

Question 14

A school collected Rs.2304 as fees from its students. If each student pays as much money as the number of students in the school, how many students were there in the school ?

Solution:

The amount paid by each student = The total number of students in the school.

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
	3

$\therefore$ The amount paid by each student = (The total number of students in the school)

= Rs.2304 = 230400 paise

$\therefore$ The total number of students in the school = $\sqrt{230400} = 10\sqrt{2304}$

$$=10\sqrt{2\times2\times2\times2\times2\times2\times2\times2\times3\times3} = 10\times2\times2\times2\times2\times3 = 480$$

There are 480 students in the school.

Question 15

By just examining the units, can you tell which of the following cannot be perfect squares?

(i) 1026
(ii) 1028
(iii) 1024
(iv) 1022
(v) 1023
(vi) 1027

Solution:

(i), (ii), (iv), (v) and (vi) cannot be whole perfect squares.

This is because square numbers do not end in the digits 2,3,7 or 8.

Question 16

Find the square roots of the perfect square 44944 by factorising.

Solution:

2	44944
3	22472
2	11236
2	5618
53	2809
	53

The prime factorization of 44944 is $2 \times 2 \times 2 \times 2 \times 53 \times 53 = (2^2 \times 2^2 \times 53^2) = (2 \times 2 \times 53)^2$

Thus the square root of 44944 is $2 \times 2 \times 53 = 212$.

Question 17

Find the square roots of the perfect square 28900 by factorising.

Solution:

2	28900
2	14450
5	7225
5	1445
17	289
	17

The prime factorization of 28900 is $2 \times 2 \times 5 \times 5 \times 17 \times 17$

Thus the square root of 28900 is $2 \times 5 \times 17 = 170$

Question 18

If $\sqrt{m} = 24$, find $2m + 1$.

Solution:

Given $\sqrt{m} = 24$

$m = 24 \times 24$

$m = 576$

$\Rightarrow 2m + 1 = 2(576) + 1 = 1152 + 1 = 1153$

Question 19

Find the square root of the following rational number: $\frac{841}{5184}$

Solution:

$\sqrt{\frac{841}{5184}} = \frac{\sqrt{841}}{\sqrt{5184}} = \frac{29}{72}$

Question 20

Find the square root of the following rational number: $\frac{625}{6889}$

Solution:

$\sqrt{\frac{625}{6889}} = \frac{\sqrt{625}}{\sqrt{6889}} = \frac{25}{83}$

Question 21

Find the square root of the rational number: $\frac{2916}{6241}$

Solution:

$\frac{\sqrt{2916}}{\sqrt{6241}} = \frac{54}{79}$

Question 22

If $\frac{2a}{\sqrt{2}} = 2\sqrt{2}$, find a.

Solution:

$\frac{2a}{\sqrt{2}} = 2\sqrt{2}$

$a = \sqrt{2} \times \sqrt{2} = 2$

Question 23

Find the squares of the following rational numbers

(i) $\frac{3}{5}$ (ii) $-\frac{2}{7}$ (iii) $-\frac{75}{100}$

Solution:

(i) $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$

(ii) $\left(-\frac{2}{7}\right)^2 = \frac{4}{49}$

(iii) $\left(-\frac{75}{100}\right)^2 = \left(-\frac{3}{4}\right)^2 = \frac{9}{16}$

Question 24

Find the smallest number that should be multiplied by 5408 to make it a perfect square.

Solution:

2	5408
2	2704
2	1352
2	676
2	338
13	169
	13

The prime factorization of 5408 is 2 × 2 × 2 × 2 × 2 × 13 × 13.
The number 2 does not have a pair and hence the smallest number that should be multiplied by 5408 to make it a perfect square is 2.

Question 25

Find the square root of 0.04.

Solution:

```
     0.  2
   ------
0 | 00 . 04
  | 00
  ------
2 |    04
  |   -04
  ------
        00
```

The square root of 0.04 is 0.2